1 The functions f and g are defined as

$$f : x \mapsto 5x - 7$$

$$g : x \mapsto \frac{5x}{x+4}$$

(a) Write down the value of x that must be excluded from any domain of g

$x = -4 \rightarrow$ since denominator of $g(x)$ cannot be zero

-4 (1)

(1)

(b) Find gf(2.6)

$f(2.6) = 5(2.6) - 7 = 6$ (1)

$gf(2.6) = g(6) = \frac{5(6)}{6+4} = \frac{30}{10} = 3$ (1)

3

(2)

(c) Solve fg(x) = 2

$fg(x) = 5\left(\frac{5x}{x+4}\right) - 7$ (1)

$= \frac{25x}{x+4} - 7$

$fg(x) = 2 = \frac{25x}{x+4} - 7$

$2x + 8 = 25x - 7x - 28$ (1)

$25x - 7x - 2x = 8 + 28$

$16x = 36$

$x = 2.25$ (1)

$x =$ 2.25

(3)

(d) Express the inverse function g^{-1} in the form $g^{-1} : x \mapsto \ldots$

$g(x) = \frac{5x}{x+4}$

Let $y = g(x)$. Find x in term of y.

$y = \frac{5x}{x+4}$

$y(x+4) = 5x$ (1)

$yx + 4y = 5x$

$4y = 5x - yx$

$4y = x(5-y)$ (1)

$x = \frac{4y}{5-y} \Rightarrow g^{-1}(x) = \frac{4x}{5-x}$ (1)

$g^{-1} : x \mapsto \frac{4x}{5-x}$

(3)

(Total for Question 1 is 9 marks)

The function g is such that $g(x) = \dfrac{4}{x+3}$ $\quad x \neq -3$

2 (c) Work out fg(2)

$f(x) = (x-4)^2$

$g(x) = \dfrac{4}{x+3}$

$fg(x) = \left(\dfrac{4}{x+3} - 4\right)^2$

$fg(2) = \left(\dfrac{4}{2+3} - 4\right)^2$ (1)

$= \left(-\dfrac{16}{5}\right)^2 = \dfrac{256}{25}$

$\dfrac{256}{25}$ (1)

(2)

(Total for Question 2 is 2 marks)

3 The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geqslant -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \ldots$

$fg(x) = (x+3)^2 - 2(x+3)$ (1)

$= x^2 + 6x + 9 - 2x - 6$

$fg(x) = x^2 + 4x + 3$ (1)

$(x+2)^2 - 4 + 3$

$fg(x) = (x+2)^2 - 1$

$fg(x) = h(x) = (x+2)^2 - 1$

Let $h(x) = y$

$y = (x+2)^2 - 1$ (1)

Find x in terms of y :

$y + 1 = (x+2)^2$

$\pm\sqrt{y+1} = x + 2$

$x = -2 \pm \sqrt{y+1}$ (1)

$\therefore h^{-1}(x) = -2 \pm \sqrt{x+1}$

equal to range of $h(x)$

since domain of $h^{-1}(x) \geq -2$,

$h^{-1}(x) = -2 + \sqrt{x+1}$ (1)

$h^{-1}(x) = -2 + \sqrt{x+1}$

(Total for Question 3 is 5 marks)

4 The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \quad \text{where } x \geqslant 1$$

same as domain of $f^{-1}(x)$

$$g(x) = 7x - 6$$

(a) Find fg(2)

$g(2) = 7(2) - 6$
$= 14 - 6$
$= 8$ (1)

$fg(2) = f(8) = 5(8)^2 - 10(8) + 7$
$= 5(64) - 80 + 7$
$= 247$ (1)

247 ....
(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

Let $y = f(x)$

$y = 5x^2 - 10x + 7$

Find x in terms of y:

$y = 5(x^2 - 2x) + 7$ (1)
$y = 5[(x-1)^2 - 1] + 7$
$y = 5(x-1)^2 - 5 + 7$ (1)
$y = 5(x-1)^2 + 2$
$y - 2 = 5(x-1)^2$
$\frac{y-2}{5} = (x-1)^2$ (1)

$\pm\sqrt{\frac{y-2}{5}} = x - 1$

$x = 1 \pm \sqrt{\frac{y-2}{5}}$

$1 - \sqrt{\frac{y-2}{5}}$ is not a solution because domain of $f^{-1}(x) \geq 1$

$x = 1 + \sqrt{\frac{y-2}{5}}$

$f^{-1}(x) = 1 + \sqrt{\frac{x-2}{5}}$ (1)

$f^{-1}(x) =$ $1 + \sqrt{\frac{x-2}{5}}$ ....
(4)

(Total for Question 4 is 6 marks)

5 The functions f and g are defined as

$\mathrm{f}(x) = x^2 + 6$
$\mathrm{g}(x) = x - 10$

(a) Find fg(3)

$\mathrm{fg}(x) = (x-10)^2 + 6$
$= (3-10)^2 + 6$ (1)
$= 55$ (1)

55
(2)

(b) Solve the equation $\mathrm{fg}(x) = \mathrm{f}(x)$
Show clear algebraic working.

$(x-10)^2 + 6 = x^2 + 6$ (1)
(1) $x^2 - 20x + 100 + 6 = x^2 + 6$
$-20x + 106 = 6$
$100 = 20x$
$x = 5$ (1)

5
(3)

The function h is defined as

$$\mathrm{h}(x) = \frac{2x - 4}{x}$$

(c) State the value of x that cannot be included in the domain of h

0 (1)
(1)

(d) Express the inverse function h^{-1} in the form $\mathrm{h}^{-1}(x) = \ldots$

Let $h(x)$ be y

$y = \frac{2x-4}{x}$

(1) $yx = 2x - 4$ — make x the subject

$yx - 2x = -4$

$x(y-2) = -4$ (1)

$x = \frac{-4}{y-2}$

$h^{-1}(x) = \frac{-4}{x-2}$ (1)

$\mathrm{h}^{-1}(x) =$ $\frac{-4}{x-2}$
(3)

(Total for Question 5 is 9 marks)

6 The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

$f(x) = x^2 - 8x + 5$

Let $f(x) = y$

$y = x^2 - 8x + 5$

$= (x-4)^2 - 16 + 5$ 1

$y = (x-4)^2 - 11$

$y + 11 = (x-4)^2$

$\pm\sqrt{y+11} = x - 4$

$x = 4 \pm \sqrt{y+11}$ 1

since domain $f^{-1}(x) \leqslant 4$,

$f^{-1}(x) = 4 - \sqrt{x+11}$ only 1

$f^{-1}(x) =$ $4 - \sqrt{x+11}$

(Total for Question 6 is 3 marks)

7 $f(x) = x^2 - 4$

$g(x) = 2x + 1$

Solve $fg(x) > 0$
Show clear algebraic working.

$fg(x) = (2x+1)^2 - 4$ (1)

$fg(x) > 0$

$(2x+1)^2 - 4 > 0$ (1)

$(2x+1)^2 > 4$

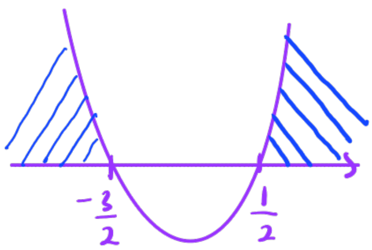
$2x+1 > \pm\sqrt{4}$

$2x+1 = 2$, $2x+1 = -2$

$x = \frac{1}{2}$, $x = -\frac{3}{2}$ (1)

$x < -\frac{3}{2}$, $x > \frac{1}{2}$ (1)

$fg(x) > 0$



$x < -\frac{3}{2}$, $x > \frac{1}{2}$ ....

(Total for Question 7 is 4 marks)

8 (b) Express the inverse function f^{-1} in the form $f^{-1}: x \mapsto \ldots$

let $f(x) = y$

$$y = \frac{2x}{x-6}$$

$y(x-6) = 2x$ 1

$yx - 6y = 2x$

$yx - 2x = 6y$

$x(y-2) = 6y$ 1

$$x = \frac{6y}{y-2}$$

$f^{-1}(x) = \dfrac{6x}{x-2}$ 1

$f^{-1}: x \mapsto$ $\dfrac{6x}{x-2}$

(3)

(Total for Question 8 is 3 marks)

9 The functions f and g are such that

$$f(x) = x + 25 \qquad g(x) = x^2 - 12x$$

The function h is such that $h(x) = fg(x)$

The domain of h is $\{x : x \leqslant 6\}$

Express the inverse function h^{-1} in the form $h^{-1}(x) = \ldots$

$h(x) = (x^2 - 12x) + 25$

$= x^2 - 12x + 25$ (1)

$= (x-6)^2 - 36 + 25$

$= (x-6)^2 - 11$ (1) $\{x : x \leqslant 6\}$

let $h(x) = y$

$y = (x-6)^2 - 11$

$y + 11 = (x-6)^2$ (1)

$\pm\sqrt{y+11} = x - 6$

$x = 6 \pm \sqrt{y+11}$

$h^{-1}(x) = 6 \pm \sqrt{x+11}$

since domain of h is $x \leqslant 6$, then $h^{-1}(x) \leqslant 6$

Hence, $h^{-1}(x) = 6 - \sqrt{x+11}$ (1)

$h^{-1}(x) =$ $6 - \sqrt{x+11}$

(Total for Question 9 is 4 marks)

10 The function g is defined as

$$g : x \mapsto 5 + 6x - x^2 \quad \text{with domain } \{x : x \geqslant 3\}$$

(a) Express the inverse function g^{-1} in the form $g^{-1} : x \mapsto \ldots$

let $g(x) = y$

$y = 5 + 6x - x^2$

$y = -(x^2 - 6x) + 5$

$y = -[(x-3)^2 - 9] + 5$ (1)

$= -(x-3)^2 + 9 + 5$

$y = 14 - (x-3)^2$ (1)

$(x-3)^2 = 14 - y$

$x - 3 = \pm\sqrt{14 - y}$

$x = 3 \pm \sqrt{14 - y}$ (1)

$g^{-1}(x) = 3 \pm \sqrt{14 - x}$

$\therefore$ since domain of $x : x \geqslant 3$,

range of $g^{-1}(x) \geqslant 3$.

Hence, $g^{-1}(x) = 3 + \sqrt{14 - x}$ (1)

$g^{-1} : x \mapsto$ $3 + \sqrt{14 - x}$

(4)

(b) State the domain of g^{-1}

(1) $x \leq 14$

(1)

(Total for Question 10 is 5 marks)

11 (b) Find $f^{-1}(x)$

let $f(x) = y$,

$$y = \frac{2}{3x-5}$$

$$3x - 5 = \frac{2}{y} \quad 1$$

$$3x = \frac{2}{y} + 5$$

$$x = \frac{2+5y}{3y} \qquad f^{-1}(x) = \frac{2+5x}{3x} \quad 1$$

$f^{-1}(x) = \frac{2+5x}{3x}$ ....

(2)

(Total for Question 11 is 2 marks)

12 The function f is such that $f(x) = \frac{k}{x}$ where $x \neq 0$ and k is an integer.

(a) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

let $f(x) = y$

$y = \frac{k}{x}$

$x = \frac{k}{y}$, $f^{-1}(x) = \frac{k}{x}$

$f^{-1}(x) =$ $\frac{k}{x}$ 1

(1)

The function g is such that $g(x) = 2 - 3x^4$ where $x \neq 0$

The function h is such that $h(x) = \frac{3x}{2 - x}$ where $x \neq 2$

(b) (i) Find g(−2)

$g(-2) = 2 - 3(-2)^4$

$= 2 - 3(16)$

$= 2 - 48 = -46$

-46 1

(1)

(ii) Express the composite function hg in the form $hg(x) = \ldots$
Give your answer in its simplest form.

$hg(x) = \frac{3(2-3x^4)}{2-(2-3x^4)}$ 1

$= \frac{6-9x^4}{3x^4}$

$= \frac{2-3x^4}{x^4}$ 1

$hg(x) =$ $\frac{2-3x^4}{x^4}$

(2)

(Total for Question 12 is 4 marks)

13 The function f is such that $f(x) = 3x^2 - 12x + 7$ where $x \leqslant 2$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

let $f(x) = y$

$y = 3x^2 - 12x + 7$

$y = 3(x^2 - 4x) + 7$ (1)

$= 3[(x-2)^2 - 4] + 7$

$= 3(x-2)^2 - 12 + 7$

$y = 3(x-2)^2 - 5$ (1)

$\frac{y+5}{3} = (x-2)^2$

$\pm\sqrt{\frac{y+5}{3}} = x - 2$

$x = 2 \pm \sqrt{\frac{y+5}{3}}$ (1)

$f^{-1}(x) = 2 \pm \sqrt{\frac{x+5}{3}}$

since domain of x of $f(x)$: $x \leq 2$,

range of $f^{-1}(x) \leq 2$

Hence, $f^{-1}(x) = 2 - \sqrt{\frac{x+5}{3}}$ (1)

$f^{-1}(x) =$ $2 - \sqrt{\frac{x+5}{3}}$

(Total for Question 13 is 4 marks)

14 The functions g and h are such that

$$g(x) = \frac{11}{2x - 5}$$

$$h(x) = x^2 + 4 \qquad x \geqslant 0$$

(a) What value of x must be excluded from any domain of g?

$2x - 5 = 0$

$x = \frac{5}{2}$

$\frac{5}{2}$ 1

(1)

(b) Solve gh(x) = 1

$gh(x) = \frac{11}{2(x^2+4)-5}$ 1

$1 = \frac{11}{2(x^2+4)-5}$

$2x^2 + 8 - 5 = 11$

$2x^2 = 8$

$x^2 = 4$ 1

$x = 2$ since $x \geqslant 0$

1

2

(3)

(Total for Question 14 is 4 marks)